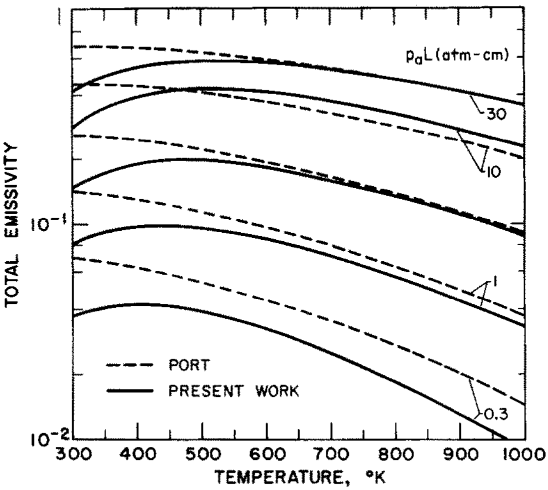

FIG. 1. Total emissivity of ammonia.

as calculated from this correlation and extrapolation procedure is shown in Fig. 1 in comparison with the early findings of Port. It should be noted that in Fig. 1 p_a stands for the ammonia partial pressure, and L for the geometric mean beam length. The agreement between the suggested values of Port and the present prediction is quite good indeed. It is a little surprising, however, to see that the agreement in the extrapolated (higher temperatures) region is better than that at 300°K, since the prediction at 300°K should be most reliable as it is purely based on the France–Williams band absorption data.

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HEAT TRANSFER PARAMETERS OF A PARALLEL PLATE HEAT EXCHANGER

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NOMENCLATURE

- a, distance between the plates through which laminar flow occurs;
- b, wall thickness;
- C_i, specific heat of fluid, i;
- g, dimensionless velocity distribution of the laminar—side fluid, $u/\bar{u}$;

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G, Graetz function;
h_i, heat transfer coefficient of fluid i;
H, heat capacity flow rate ratio, C_2W_2/C_1W_1;
k_1, thermal conductivity of the laminar—side fluid;
K, relative thermal resistance of the common wall and turbulent side convection;
k_w, wall thermal conductivity;
l, axial length measured from the laminar side inlet;
L, overall length of the exchanger;
Nu_1, laminar side Nusselt number, $2a\,h_1/k$;
Nu_1°, overall Nusselt number, $2a\,U_1/k$;
N_1, normalised laminar-side Nusselt number;
Pe, laminar-side Péclét number, $2a\,\bar{u}/\alpha$;
t_i, temperature of fluid i;
$t_{1,0}$, laminar side inlet temperature;
$t_{2,0}$, turbulent side inlet temperature;
u, local axial laminar side fluid velocity;
$\bar{u}$, average laminar side fluid velocity;
U_1, overall heat transfer coefficient referred to laminar side;
W_i, mass rate of flow of fluid i;
x, dimensionless transverse position, y/a;
y, transverse position measured from the bottom plate;
z, dimensionless axial position $(2/Pe)(l/a)$;
Z, dimensionless heat exchanger length $(2/Pe)(L/a)$;
α, thermal diffusivity of laminar side fluid;
ε, heat exchanger effectiveness;
ξ_i, dimensionless temperature of fluid i, $(t_i - t_{2,0})/(t_{i,0} - t_{2,0})$;
Δ, additional heat exchanger length;
$\ldots(\infty)$, fully developed values.

Subscripts
1, laminar side;
2, turbulent side;
∞, fully developed value.

THE DESIGN of heat exchangers, is customarily based on the assumption of uniform heat-transfer coefficient along the length of the exchanger, irrespective of the boundary conditions. This assumption is reasonably valid only for turbulent flow of fluids [1, 2]. But for fluids in laminar flow the heat-transfer coefficients become sensitive to the actual boundary conditions and may not be sufficiently uniform along the length of the exchanger. Also thermal entrance regions for laminar flow can be significant [1, 3–5].

Stein and Sastri [6] recently presented a detailed analysis of heat exchanger with laminar tube-side and turbulent shell-side flows as a new extension of the classical Graetz problem and reported various quantities relating to cocurrent and countercurrent flows. They assumed uniform heat transfer coefficient on the shell-side and solved the two-dimensional energy equation for the tube-side fluid and showed that predictions can be made by use of the actual fully developed heat transfer coefficient and an effective heat exchanger length and that both of these quantities depend on the operating conditions of the exchanger.

The present note applies the above analysis to a parallel plate heat exchanger with laminar flow on one side and turbulent flow on the other. The fully developed Nusselt number and the thermal entrance length are given as functions of operating parameters for both cocurrent and countercurrent flows.

ANALYSIS

A schematic of the parallel plate exchanger is shown in Fig. 1. Assuming a constant heat-transfer coefficient on the turbulent side, the appropriate laminar side energy equation is written in dimensionless form as

$$\frac{\partial^2 \xi_1}{\partial x^2} = g(x)\frac{\partial \xi_1}{\partial z}, \qquad \xi_1(x, z): 0 \leqslant x \leqslant 1,\quad 0 \leqslant z \leqslant Z \tag{1}$$

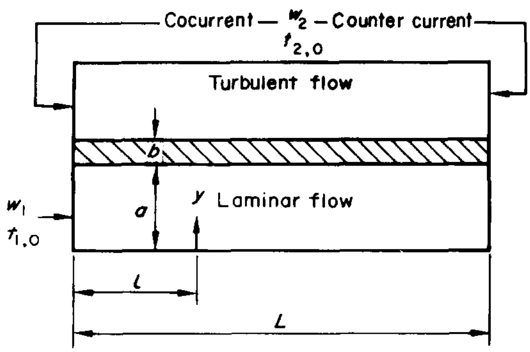

FIG. 1. Schematic diagram of a parallel plate heat exchanger.

where

$$g(x) = 6x(1 - x). \tag{2}$$

Here the equivalent Graetz function $G(\lambda, x)$ is assumed to be of the form

$$G(\lambda, x) = \sum_{n=1}^{\infty} A_n(\lambda)x^n \tag{3}$$

with the recurrence relation given by

$$A_0 = 1,\ A_1 = 0,\ A_2 = 0$$

and

$$A_n = \frac{6\lambda}{n(n-1)}[A_{n-4} - A_{n-3}] \text{ for } n \geqslant 3. \tag{4}$$

NUSSELT NUMBERS

It can be shown that for sufficiently large z, the laminar side Nusselt number can be given by

$$Nu_1(\infty) = 2H\lambda_1/(H + \delta - KH\lambda_1) \quad (5)$$

where λ_1 is the first order eigen value of the characteristic equation and the corresponding overall Nusselt number is given by

$$Nu_1^\circ(\infty) = 2H\lambda_1/(H + \delta). \quad (6)$$

For the special case of $\delta = -1$ and $H = 1$, it is known that

$$Nu_1(\infty) = 70/13 = 5{\cdot}385 \quad (7)$$

which corresponds to the boundary condition of uniform wall heat flux. The corresponding $Nu_1^\circ(\infty)$ is given by

$$Nu_1^\circ(\infty) = 70/(13 + 35K). \quad (8)$$

It may be noted that the uniform wall temperature boundary condition is attained as $H \to \infty$ and $K \to 0$ (or $KH \to 1$). For this case λ_1 is found to be 2·4303 and the corresponding

$$Nu_1(\infty) = Nu_1^\circ(\infty) = 2\lambda_1 = 4{\cdot}8606.$$

ADDITIONAL HEAT EXCHANGER LENGTH

The traditional definition of NTU may be modified by writing

$$NTU = \tfrac{1}{2} Nu_1^\circ(\infty)\,[Z + \Delta] \quad (9)$$

for this geometry, where Δ is the appropriate additional heat exchanger length, which takes into account the effects of thermal entrance regions. Thus, we obtain the relations for Δ as follows: (except for $\delta = -1$, $H = 1$)

$$\Delta = -\frac{1}{\lambda_1}\ln\left[-\frac{H+\delta}{H}\sum_{n=1}^{\infty} B_n \exp\{-(\lambda_n - \lambda_1)Z\}\right] \quad (10)$$

and

$$\Delta_\infty = -\frac{1}{\lambda_1}\ln\left[-\left(\frac{H+\delta}{H}\right)B_1\right]. \quad (11)$$

For the case of uniform wall heat flux ($\delta = -1$, $H = 1$),

$$\Delta = \Delta_\infty + \left(\frac{26 + 70K}{70}\right)\sum_{n=1} B_n \exp(-\lambda_n Z) \quad (12)$$

with

$$\Delta_\infty = \frac{4454}{3 \times 4 \times 11 \times 35 \times (26 + 70K)}. \quad (13)$$

Δ is, in general, a function of H, K and the mode of operation and the length of the exchanger Z. However, in most cases of practical interest where the effectiveness is greater than 0·5, Z is sufficiently large such that Δ_∞ would be sufficient for most applications [3].

RESULTS AND DISCUSSION

The factors that are directly related to the overall heat transfer rates are the laminar side fully developed Nusselt number $Nu_1(\infty)$ and the additional heat exchanger length Δ_∞.

In Fig. 2, $Nu_1(\infty)$ is normalised with respect to the value corresponding to the case of uniform wall heat flux and shown as a function of H and K. The normalized value corresponding to isothermal wall is about 0·83 and is shown in Fig. 2. The behaviour is qualitatively identical to that found with other heat exchanger analyses [3, 4, 6]. In general, it is observed that operating conditions have significant effect on laminar side heat transfer coefficients $Nu_1(\infty)$ is smaller for cocurrent flow than for counter-current

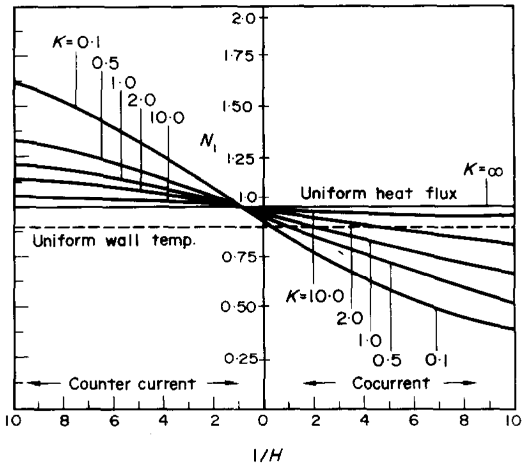

FIG. 2. Variation of normalized Nusselt number with operating parameters.

flow. In cocurrent flow, the fully developed coefficients are never larger than the value corresponding to the uniform wall heat flux boundary condition. On the other hand, in countercurrent flow, fully developed coefficients are never smaller than the value corresponding to the uniform wall temperature boundary condition, but can be significantly larger than the value corresponding to the boundary condition of uniform heat flux.

Figure 3 shows the dependence of Δ_∞ on the operating parameters. It is seen that the values of Δ_∞ for cocurrent flow are greater than for countercurrent flow, indicating that thermal entrance region is more significant with cocurrent flow than with counter flow.

For values of K less than about 1·0 and for given H, Fig. 2 shows that the order of magnitude of error in $Nu_1(\infty)$ of the traditional design formulae is the same for both flow arrangements. However, the error can be greatly magnified for countercurrent flow because Δ_∞ is much smaller as can

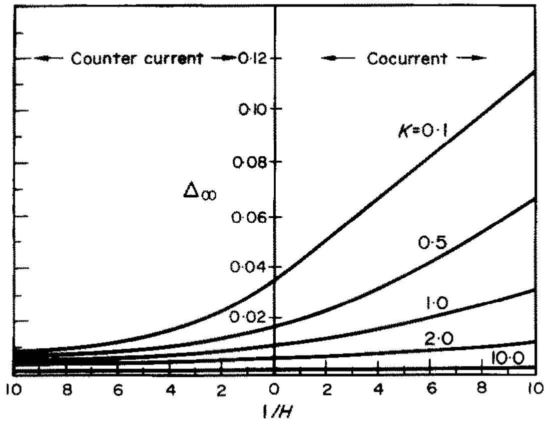

FIG. 3. Variation of additional heat exchanger length with operating parameters.

be seen in Fig. 3. This is also evident from equation (9) in which NTU is given as a product of $Nu_1(\infty)$ and Δ. This influence is even more significant for decreasing values of H.

Tables 1 and 2 give the computed quantities for cocurrent and countercurrent flows respectively. In each table are shown the laminar side fully developed Nusselt number, the additional heat exchanger length Δ_∞, Z^* and ε^*. Z^* is the value of Z at which $\Delta = \Delta_\infty$ ($\Delta = 0{\cdot}95\,\Delta_\infty$) as defined in [6] and ε^* is the corresponding effectiveness. Heat exchanger computations with $\Delta = \Delta_\infty$ require that $Z \geqslant Z^*$ or $\varepsilon \geqslant \varepsilon^*$.

These latter conditions are satisfied as can be seen in the Tables 1 and 2 so that the use of Δ_∞ is justified for most practical applications.

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Table 1. Quantities related to the cocurrent flow.

H	$Nu_1^\circ(\infty)$	$Nu_1(\infty)$	Δ_∞	Z^*	ε^*
$K = 0·1$					
0·1	1·912	2·113	0·1143	0·109	0·895
0·5	3·440	4·155	0·0502	0·138	0·616
1·0	3·706	4·549	0·0423	0·134	0·474
2·0	3·842	4·754	0·0387	0·130	0·381
10·0	3·948	4·920	0·0359	0·127	0·294
$K = 0·5$					
0·1	1·646	2·797	0·0658	0·201	0·906
0·5	1·154	4·669	0·0235	0·155	0·435
1·0	2·204	4·913	0·0205	0·144	0·301
2·0	2·228	5·033	0·0192	0·138	0·229
10·0	2·248	5·128	0·0182	0·134	0·169
$K = 1·0$					
0·1	1·282	3·563	0·0309	0·213	0·816
0·5	1·422	4·929	0·0132	0·124	0·251
1·0	1·436	5·085	0·0120	0·115	0·164
2·0	1·442	5·162	0·0115	0·110	0·121
10·0	1·446	5·122	0·0111	0·106	0·088
$K = 2·0$					
0·1	0·812	4·347	0·1140	0·114	0·420
0·5	0·836	5·124	0·0068	0·059	0·076
1·0	0·840	5·213	0·0065	0·053	0·047
2·0	0·840	5·256	0·0063	0·051	0·034
10·0	0·842	5·291	0·0062	0·049	0·024
$K = 10·0$					
0·1	0·192	5·166	0·0015	0·005	0·005
0·5	0·192	5·326	0·0014	0·005	0·001
1·0	0·192	5·346	0·0014	0·005	0·001
2·0	0·192	5·356	0·0014	0·005	0·001
10·0	0·192	5·363	0·0013	0·005	0·001

Table 2. Quantities related to the countercurrent flow

H	$Nu_1^\circ(\infty)$	$Nu_1(\infty)$	Δ_∞	Z^*	ε^*
$K = 0·1$					
0·1	6·064	8·703	0·0072	0·039	0·737
0·5	4·500	5·807	0·0242	0·105	0·399
1·0	4·242	5·385	0·0292	0·116	0·233
2·0	4·110	5·173	0·0321	0·121	0·252
10·0	4·022	5·004	0·0346	0·125	0·267
$K = 0·5$					
0·1	2·566	7·152	0·0063	0·053	0·516
0·5	2·336	5·610	0·0140	0·111	0·237
1·0	2·295	5·385	0·0158	0·122	0·135
2·0	2·274	5·269	0·0168	0·127	0·144
10·0	2·256	5·176	0·0177	0·131	0·152
$K = 1·0$					
0·1	1·532	6·542	0·0052	0·045	0·307
0·5	1·468	5·528	0·0092	0·089	0·128
1·0	1·458	5·385	0·01	0·097	0·071
2·0	1·452	5·311	0·0105	0·101	0·075
10·0	1·448	5·252	0·0109	0·105	0·079
$K = 2·0$					
0·1	0·858	6·080	0·0038	0·017	0·074
0·5	0·846	5·468	0·0055	0·040	0·036
1·0	0·843	5·385	0·0058	0·044	0·019
2·0	0·842	5·342	0·0060	0·047	0·021
10·0	0·842	5·308	0·0061	0·048	0·022
$K = 10·0$					
0·1	0·194	5·554	0·0012	0·005	0·005
0·5	0·192	5·404	0·0013	0·005	0·001
1·0	0·193	5·385	0·0013	0·005	0·001
2·0	0·192	5·375	0·0013	0·005	0·000
10·00	0·192	5·367	0·0013	0·005	0·000

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